Compute

$$1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots$$

Compute

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$$1+2x+3x^2+4x^3+5x^4+\cdots$$

$$\int 1+2x+3x^2+4x^3+5x^4+\cdots dx$$

$$= C+x+x^2+x^3+x^4+x^5+\cdots$$

$$= C-1+1+x+x^2+x^3+\cdots$$

$$= C-1+\sum_{n=0}^{\infty} x^n$$

$$= C-1+\frac{1}{1-x}$$

$f(x)=e^x$

$f'(x)=e^x$

$f^{(n)}(x)=e^x$

$$f(x)=\frac{d}{dx}\left(C-1+\frac{1}{1-x}\right)$$

$$= -\frac{1}{(1-x)^2}(-1)$$

$$= \frac{1}{(1-x)^2}$$

$$\therefore\ 1+2x+3x^2+4x^3+5x^4+\cdots = \frac{1}{(1-x)^2}$$